

# A Signal-Processing Framework for Inverse Rendering

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# Photorealistic Rendering

## Geometry



70's, 80's: Splines  
90's: Range Data



## Materials/Lighting

(Texture Reflectance[BRDF]  
Lighting)

*Realistic input models*

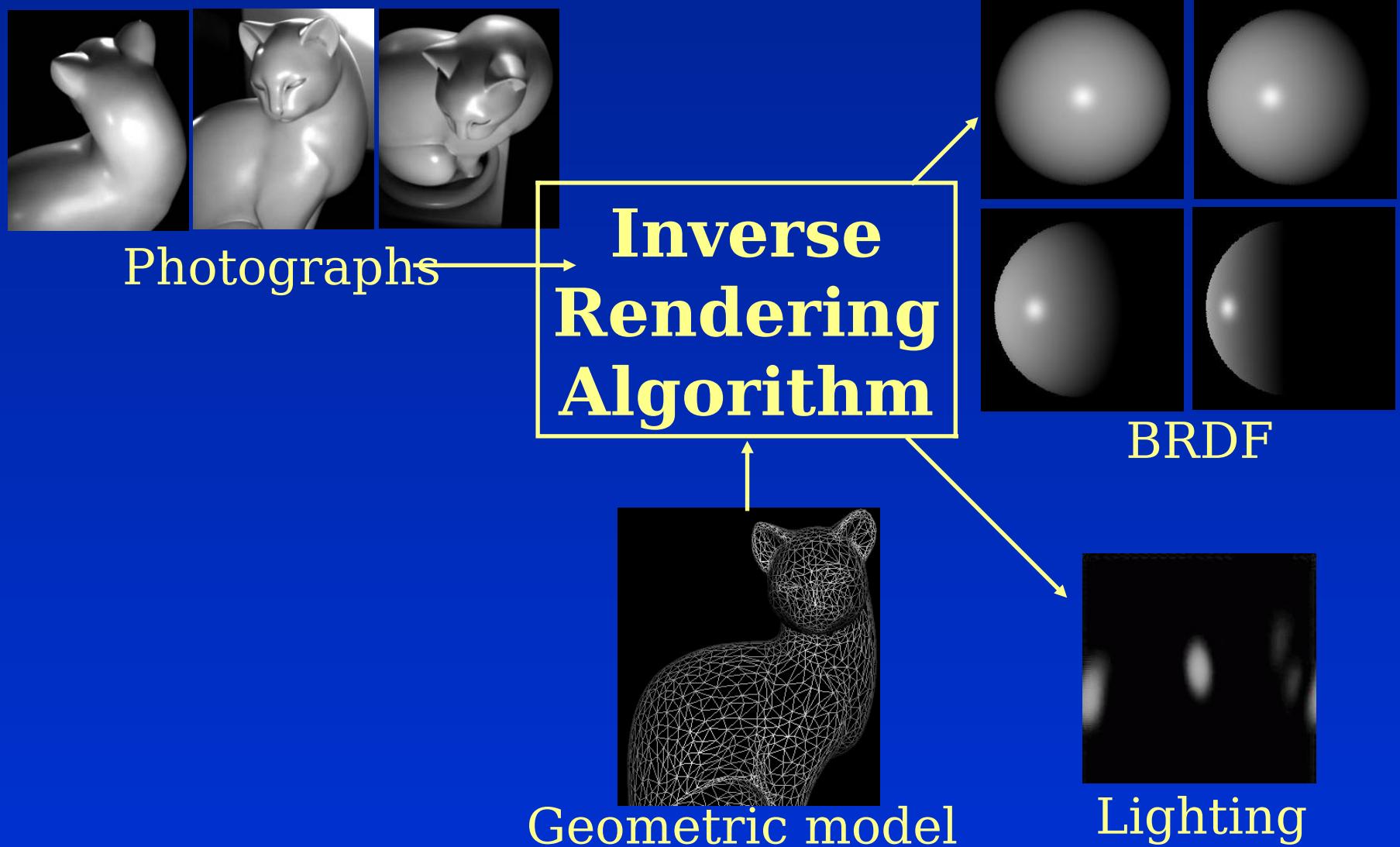
## Rendering Algorithm



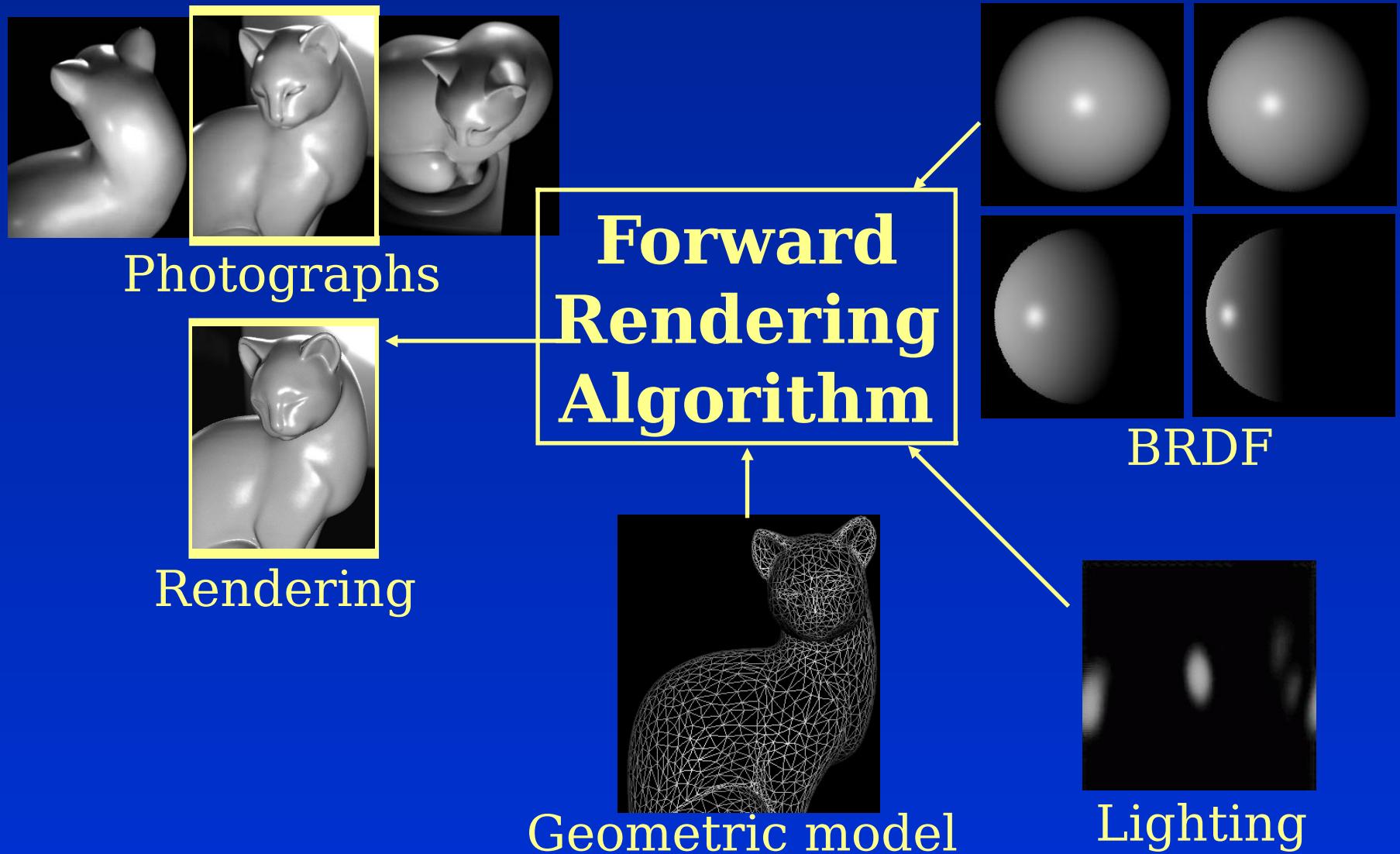
Arnold Renderer: Marcos Fajardo

80's, 90's: Physically based

# Flowchart



# Flowchart

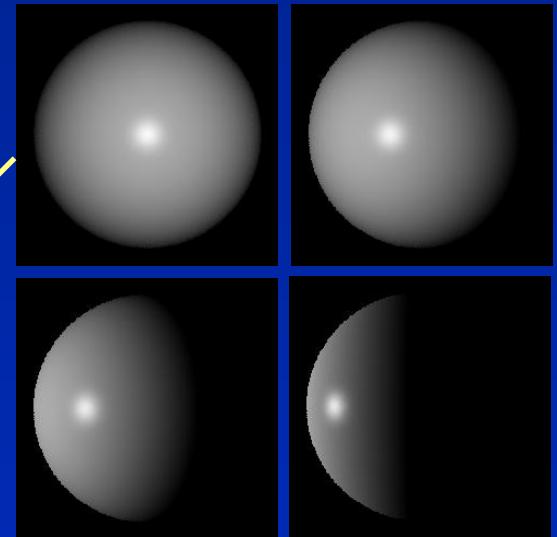


# Flowchart



Photographs

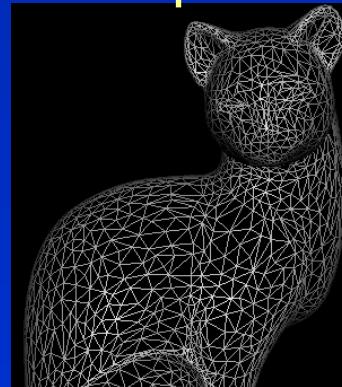
**Forward  
Rendering  
Algorithm**



BRDF



Rendering



Geometric model

Novel lighting

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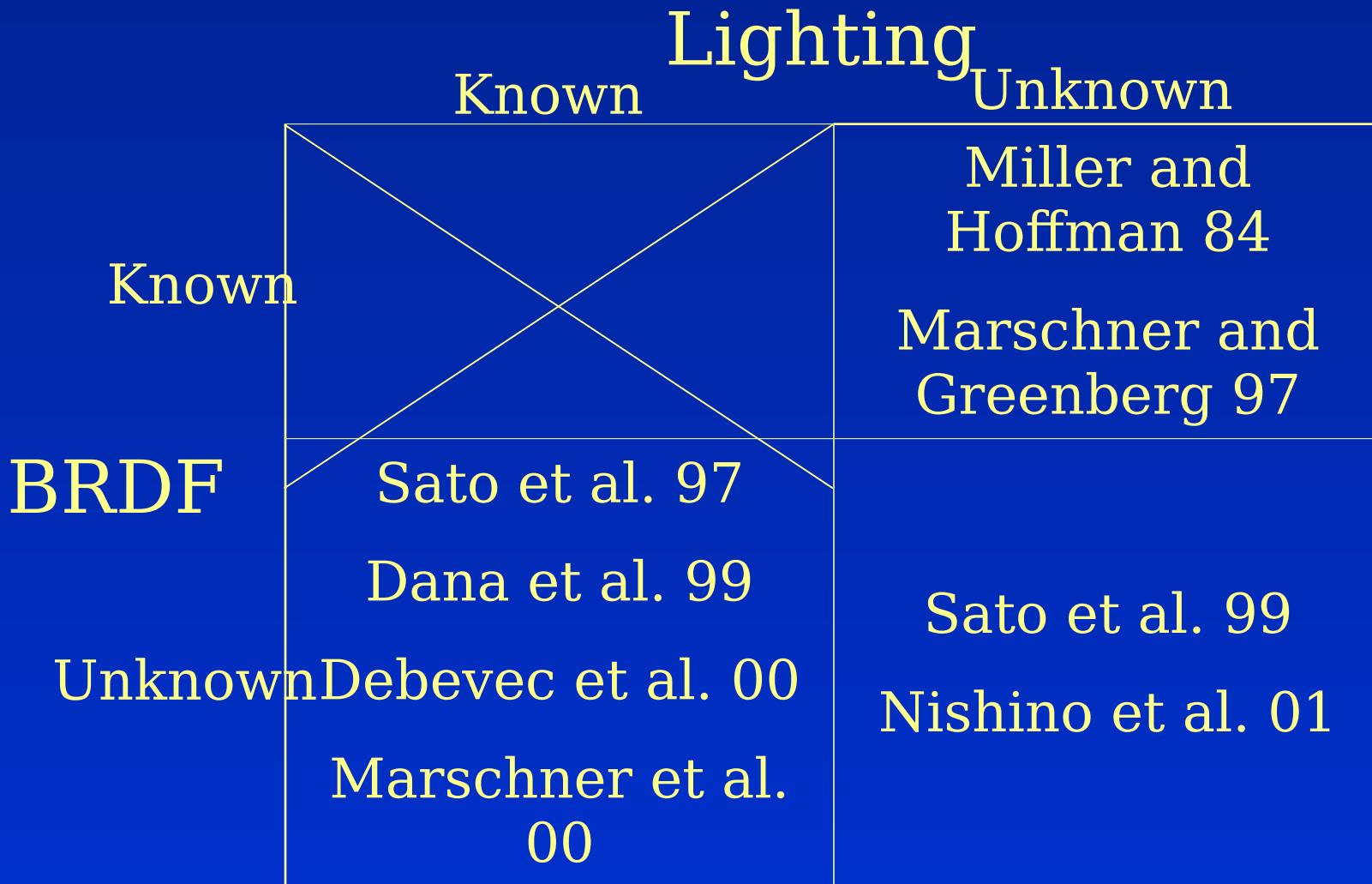
# Assumptions

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- Known geometry
- Distant illumination
- Homogenous isotropic materials
- Convex curved surfaces: no shadows, interreflection

Later, practical algorithms: relax some assumptions

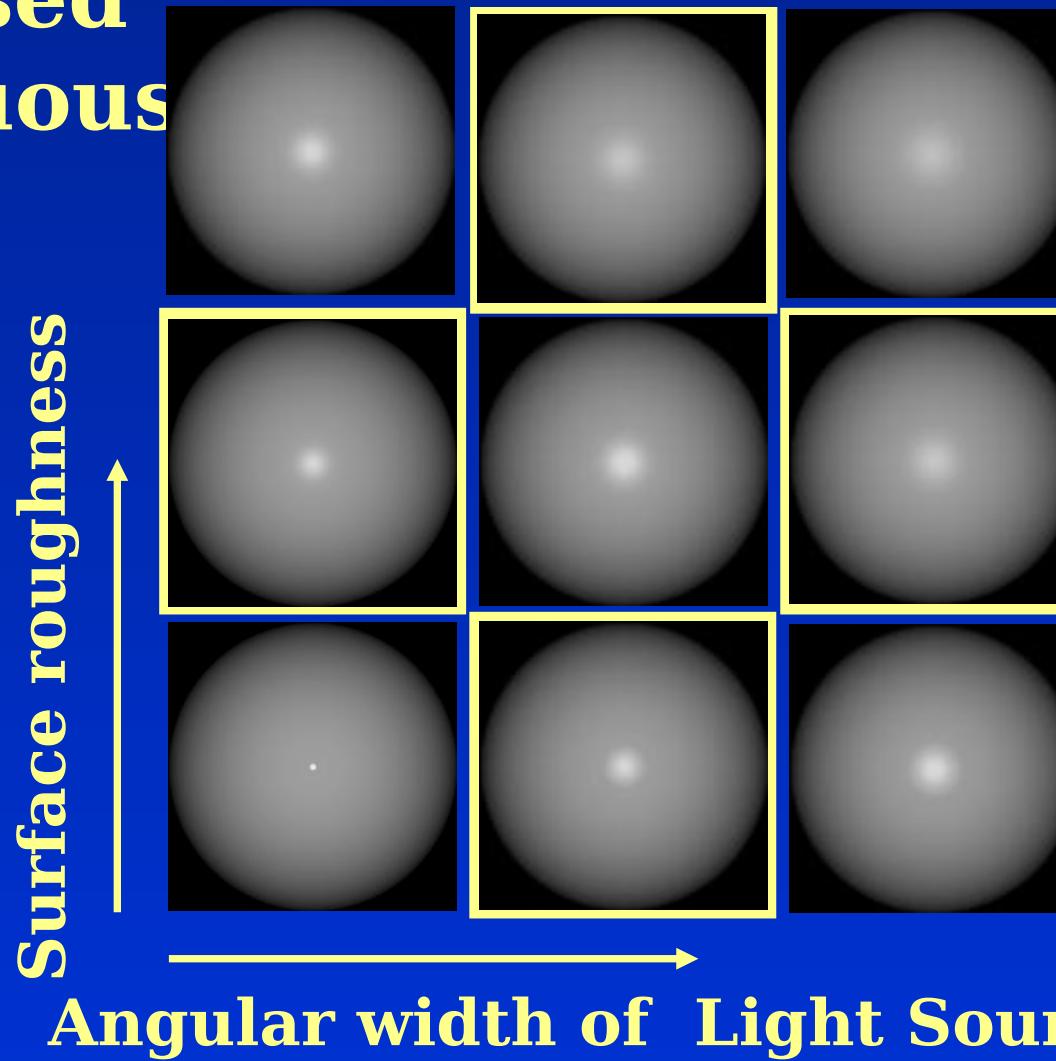
# Inverse Rendering



Textures are a third axis

# Inverse Problems: Difficulties

Ill-posed  
ambiguous



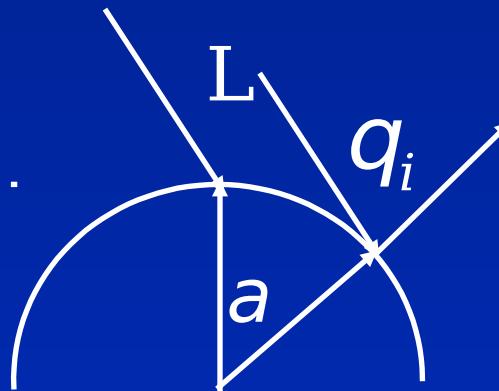
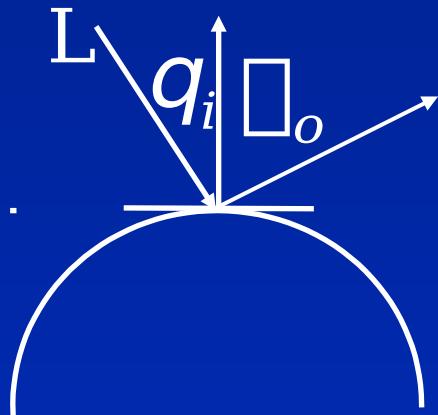
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# Contributions

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1. Formalize reflection as convolution
2. Signal-processing framework
3. Analyze well-posedness of inverse problems
4. Practical algorithms

# Reflection as Convolution (2D)

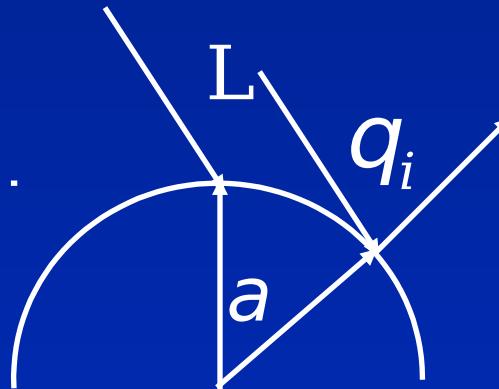
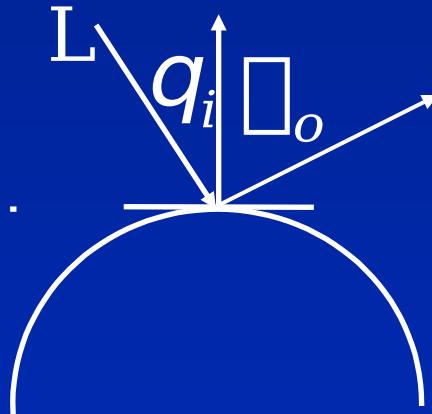


$$B(\theta_o) = \int_{\pi/2}^{\pi/2} L(\theta_i) \rho(\theta_i, \theta_o) d\theta_i$$

Reflected Light Field Lighting BRDF

$$B(\alpha, \theta_o) = \int_{\pi/2}^{\pi/2} L(\alpha + \theta_i) \rho(\theta_i, \theta_o) d\theta_i$$

# Reflection as Convolution (2D)



$$B(\alpha, \theta_o) = \int_{\pi/2}^{\pi/2} L(\alpha + \theta_i) \rho(\theta_i, \theta_o) d\theta_i$$

$$B = L \otimes \rho$$

Fourier analysis

$$B_{l,p} = 2\pi L_l \rho_{l,p}$$

Spatial: integral

Frequency: product

# Spherical Harmonic Analysis

2D:

$$B(\alpha, \theta_o) = \int_{\pi/2}^{\pi/2} L(\alpha + \theta_i) \rho(\theta_i, \theta_o) d\theta_i$$

$$B_{l,p} = 2\pi L_l \rho_{l,p}$$

3D:

$$B(\alpha, \beta, \theta_o, \phi_o) = \int_0^{\pi} \int_0^{2\pi} L(R_{\alpha, \beta}[\theta_i, \phi_i]) \rho(\theta_i, \phi_i, \theta_o, \phi_o) d\theta_i d\phi_i$$

$$B_{lm, pq} = \Lambda_l L_{lm} \rho_{lq, pq}$$

# Insights: Signal Processing

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Signal processing framework for reflection

- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

# Insights: Signal Processing

Signal processing framework for reflection

- Light is the signal
- BRDF is the filter

Filter is ~~Reflection on a curved surface is~~ Output ~~Surface~~ Signal convolution  
Mirror BRDF : Image = Lighting  
[Miller and Hoffman 84]

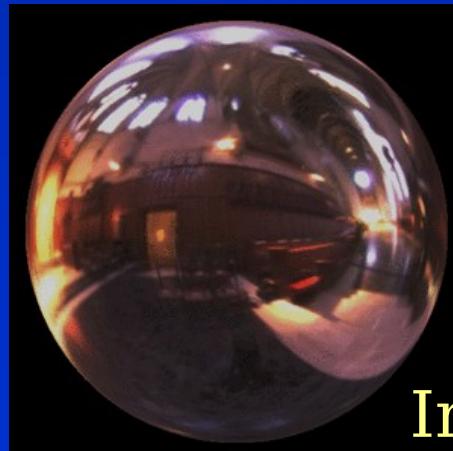


Image courtesy Paul Debevec

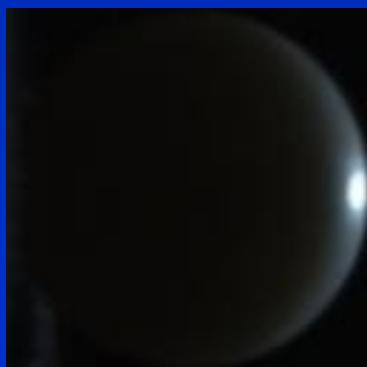
# Insights: Signal Processing

Signal processing framework for reflection

- Light is the signal
- BRDF is the filter

signal is ~~Reflection on a curved surface is~~  
~~Delta function on a curved surface is~~  
~~Output after convolution~~  
Point Light Source : Images = BRDF

[Marschner et al. 00]



# Phong, Microfacet Models

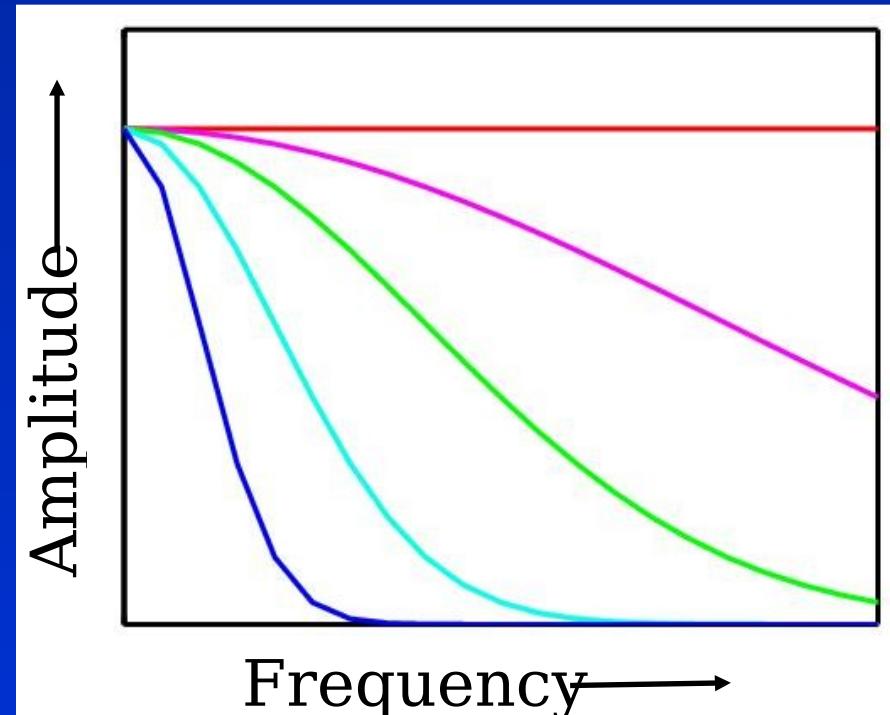


Mirror



Roughness

Illumination estimation  
ill-posed for rough surfaces



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# Inverse Lighting

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Given:  $B, \rho$  find  $L$

$$B = L \otimes \rho$$

$$B_{lm, pq} = \Lambda_l L_{lm} \rho_{lq, pq}$$

$$L_{lm} = \frac{1}{\Lambda_l} \frac{B_{lm, pq}}{\rho_{lq, pq}}$$

Well-posed unless denominator vanishes

- BRDF should contain high frequencies : Sharp highlights
- Diffuse reflectors low pass filters: Inverse lighting will need

# Inverse BRDF

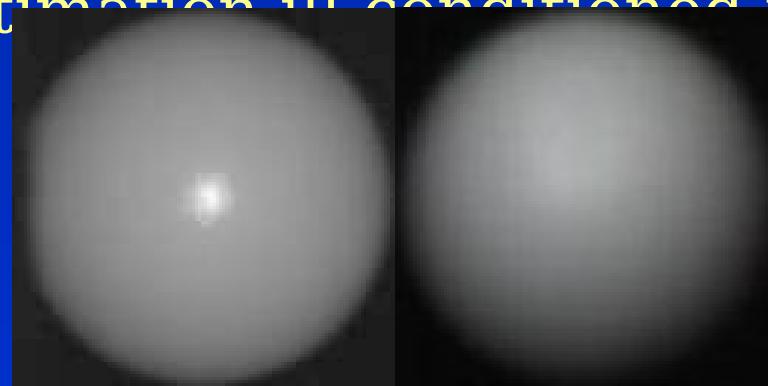
Given:  $B, L$  find  $\rho$

$$\rho_{lq, pq} = \frac{1}{\Lambda_l} \frac{B_{lm, pq}}{L_{lm}}$$

Well-posed unless  $L_{lm}$  vanishes

- Lighting should have sharp features (point sources, edges)
- BRDF estimation ill-conditioned for soft lighting

**Directio  
nal  
Source**



**Area source  
Same BRDF**

# Factoring the Light Field

Given:  $B$  find  $L$  and  $\rho$

$$B = L \otimes \rho$$

↓      ↓      ↓

4D    2D    3D

More knowns (4D)  
than unknowns (2D/3D)

Light Field can be factored

- Up to global scale factor
- Assumes reciprocity of BRDF
- Can be ill-conditioned
- Analytic formula in paper

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# Practical Issues

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- Incomplete sparse data (few photographs)  
Difficult to compute frequency spectra
- Concavities: Self Shadowing and  
Interreflection
- Spatially varying BRDFs: Textures

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# Practical Issues

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- Incomplete sparse data (few photographs)  
Difficult to compute frequency spectra
- Concavities: Self Shadowing and  
Interreflection
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Issues can be addressed; can derive practical  
algorithms

Dual spatial (angular) and frequency-space  
representation

Simple extensions for shadowing, textures

# Algorithm Validation

Photograph



“True” values

$K_d$	0.91
$K_s$	0.09
$\mu$	1.85
$\sigma$	0.13

# Algorithm Validation

Photograph



Renderings



Image RMS  
error 5%

Known lighting      Unknown lighting

“True” values

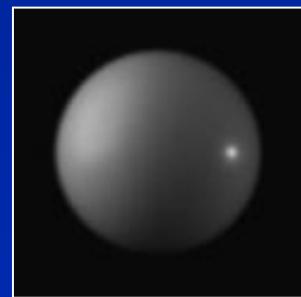
$K_d$	0.91	0.89	0.87
$K_s$	0.09	0.11	0.13
$\mu$	1.85	1.78	1.48
$\sigma$	0.13	0.12	.14

# Inverse BRDF: Spheres

Photographs



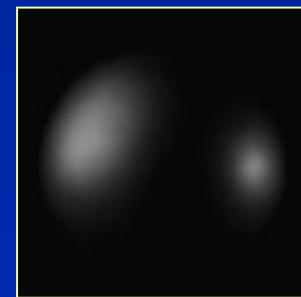
Bronze



Delrin



Paint



Rough Steel

Renderings  
(Recovered  
BRDF)



# Complex Geometry



**3 photographs of a sculpture**

- **Complex unknown illumination**
- **Geometry known**
- **Estimate microfacet BRDF *and* distant lighting**

# Comparison



Photograph



Rendering

# New View, Lighting

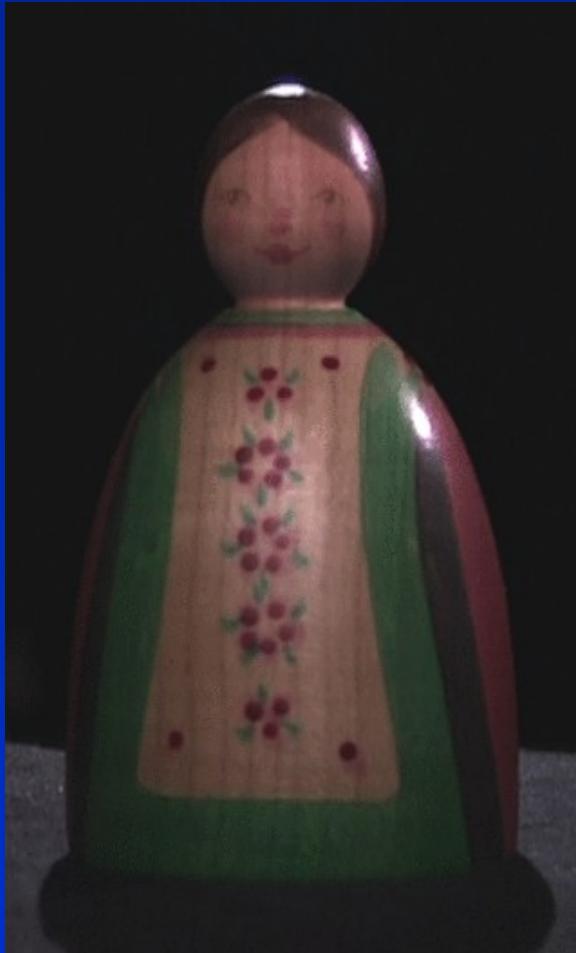


Photograph



Rendering

# Textured Objects



Photograph



Rendering

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# Summary

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- Reflection as convolution
- Signal-processing framework
- Formal study of inverse rendering
- Practical algorithms

# Implications and Future Work

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- Frequency space analysis of reflection
- Well-posedness of inverse problems
  - Perception, human vision
  - Forward rendering [Friday]
- Complex uncontrolled illumination

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# Acknowledgements

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- Marc Levoy
- Szymon Rusinkiewicz
- Steve Marschner

- John Parisenti, Jean Gleason
- Scanned cat sculpture is “Serenity” by Sue Dawes
- Hodgson-Reed Stanford Graduate Fellowship
- NSF ITR grant #0085864: “Interacting with the Visual World”

Paper Website:

<http://graphics.stanford.edu/papers/invrend>

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# The End

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# Related Work

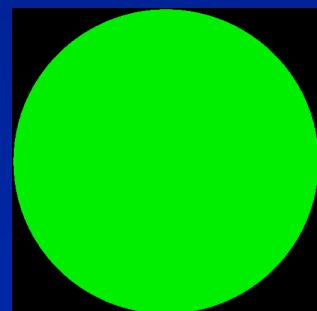
- Qualitative observation of reflection as convolution: Miller & Hoffman 84, Greene 86, Cabral et al. 87,99
- Reflection as frequency-space operator: D'Zmura 91
- Lambertian reflection is convolution: Basri Jacobs 01

## Our Contributions

- Explicitly derive frequency-space convolution formula
- Formal Quantitative Analysis in General 3D Case

# Spherical Harmonics (3D)

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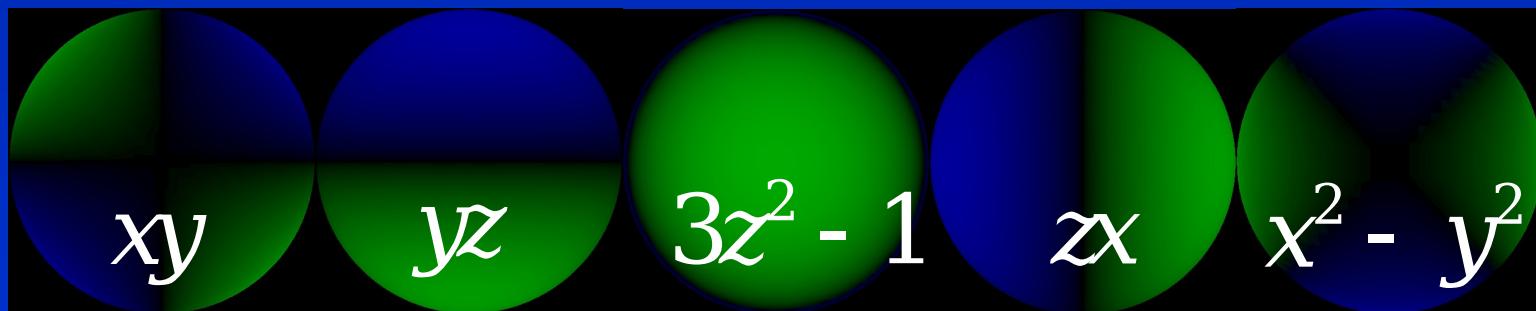


$$Y_{lm}(\theta, \phi)$$

1



2



-2

-1

0

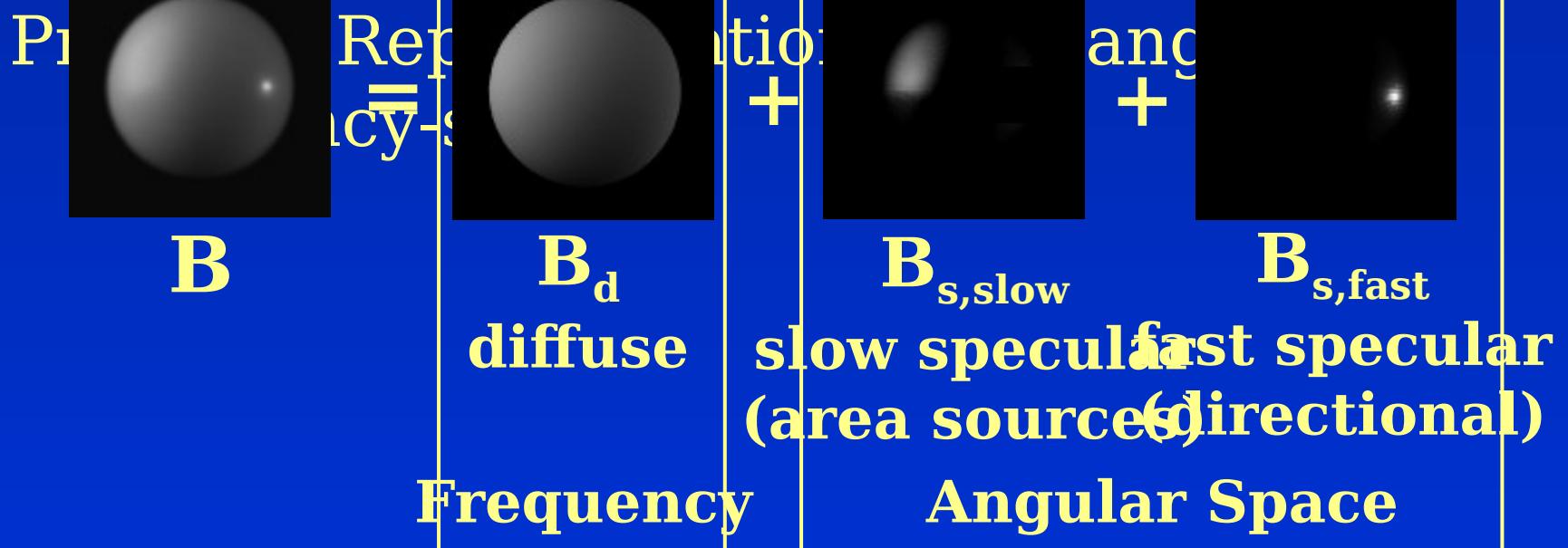
1

2

# Dual Representation

Diffuse BRDF: Filter width small in frequency domain

Specular: Filter width small in spatial (angular) domain

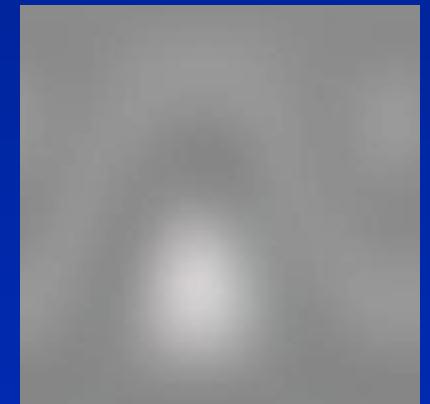
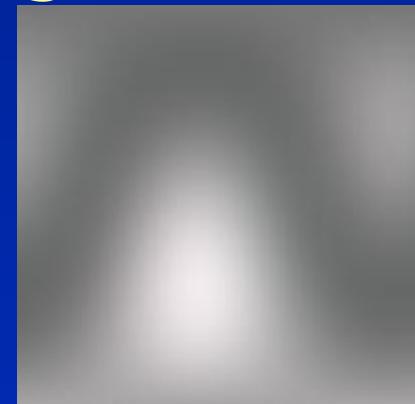


# Inverse Lambertian

True Lighting Sum  $l=2$



Sum  $l=4$



Mirror



Teflon

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# Other Papers

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- Linked to from website for this paper
  - <http://graphics.stanford.edu/papers/invrend/>
- Theory
  - Flatland or 2D using Fourier analysis [SPIE 01]
  - Lambertian: radiance from irradiance [JOSA 01]
- Application to other areas
  - Forward Rendering (Friday) [SIGGRAPH 01]
  - Lighting variability object recognition [CVPR 01]